Join Processing:

\( S = \{S\} \) tuples, \( ISI \) pages \\
\( R = \{R\} \) tuples, \( IRI \) pages \\
\( IRI < ISI \) \\
M pages of memory

Types of joins

Nested Loops

\[
\text{for } s \text{ in } S \\
\quad \text{for } r \text{ in } R \\
\quad \quad \text{if } \text{pred}(s,r) \text{ output } s \text{ join } r
\]

Does it matter which is inner, outer? (yes) \\
\( \{S\} \ast \{R\} \) compares in either case

suppose \( ISI = 4, IRI = 2, M=3, \text{LRU} \)

\[
\text{S inner } = 2 + 4 + 4 = 10 \text{ pages read} \\
\text{R inner } = 4 + 2 = 6 \text{ pages read}
\]

w/out cache: \( ISI + \{S\} \ast IRI \) i/o s

Index Nested Loops (Index on R)

\[
\text{for } s \text{ in } S \\
\quad \text{find matches in } R
\]

\( ISI + \{S\} \) i/o
Block Nested Loops

\[
B = \text{block size}
\]
\[
\text{while (not at end of S)}
\]
\[
S' = \text{read B records from S}
\]
\[
\text{for } r \text{ in } R
\]
\[
\text{for } s \text{ in } S'
\]
\[
\text{if pred}(s, r) \text{ output } s \text{ join } r
\]

Can think of different patterns off access to S & R -- in terms of blocks. Can also reverse which is "inner" and "outer" on the fly. Why might we want to do that? (Streaming) Sometimes called "pipelined"

(Sort)Merge (simple case -- in memory)  
\[1 < 2 < 3 \]
Sort R, Sort S
(merge)
IRI + ISI i/os
7 9

Hash (simple, in memory)

Build hash on R, probe with S
IRI + ISI i/os

If IRI > M, read in M pages of R, probe with S
IRI + ISI * ceil (IRI / M) i/os

Pipelined Hash
As tuples of R arrive, add to hash on R, probe into hash on S
As tuples of S arrive, add to hash on S, probe into hash on R

Why?

(Break)
Postgres demo
Shapiro: (Gossip about paper)

What's this paper about?
(Join algorithms for two relations when size of either relation exceeds available RAM)

Equality joins only

What's the big takeaway?
(hash join outperforms sort-merge join)

Always?
(at least, if you have to sort the relations)

Why is this only for "large" memories?
(Requires memory equal to sqrt(S), where S is the larger relation)
\[ M \geq \sqrt{|S|} \geq \sqrt{|R|} \]

How do these external algorithms work?

2 phases
- Phase 1: partition the relation into (sorted/hashed) runs
- Phase 2: join the partitions

Sort-merge:
  - phase 1
    - repeat until done:
      - read a run of S
      - sort
      - write out
    - repeat until done:
      - read a run of R
      - sort
      - write out

  - phase 2

begin reading from each run of R and S (requires one block of memory for each run)
join R and S as they appear

Example:

\[ R = 1, 4, 3, 6, 9, 14, 1, 7, 11 \]
\[ S = 2, 3, 7, 12, 9, 8, 4, 15, 6 \]

\[ R1 = 1, 4, 3 \quad R2 = 6, 9, 14 \quad R3 = 1, 7, 11, \text{ etc} \]
run size = 3
R1 R2 R3 S1 S2 S3
1< 6< 1< 2< 8< 4<
3 9 7 3 9 6
4 14 11 7 12 15

1 6< 1 2< 8< 4<
3< 9 7< 3 9 6
4 14 11 7 12 15

1 6< 1 2< 8< 4<
3< 9 7< 3 9 6
4 14 11 7 12 15

output 3

1 6< 1 2< 8< 4<
3 9 7< 3 9 6
4< 14 11 7< 12 15

output 4

1 6< 1 2< 8< 4
3 9 7< 3 9 6<
4 14 11 7< 12 15

output 6

1 6 1 2 8< 4
3 9< 7< 3 9 6
4 14 11 7< 12 15<

output 7 ...

How do I pick the run size?
(Make it as big as possible, to minimize the number of runs. Runs can be at most 
$M \geq \sqrt{|S|}$ to sort in memory.)

Claim: Can't be more than $M$ runs. Why?

Suppose we set the length of a run to $\sqrt{|S|}$

Then, there will be $|S|/\sqrt{|S|} = \sqrt{|S|}$ runs, which is good b/c $M > \sqrt{|S|}$, and
need one one page of memory for each run to do merge concurrently
Paper confusingly claims if you have $M = \sqrt{|S|}$ memory, runs will be size $2 \times \sqrt{|S|}$

(Where does 2 come from. Using "selection replacement tree" -- idea is that you store values in a heap, read in a new value whenever you output an old value. Average run length will be $2 \times$ size of memory). See Knuth.

$$
23 \\
45 \\
64 \\
12 \\
19 \\
82 \\
97 \\
44
$$

$M = 3$

$$
23 \\
23 45 \\
23 45 64
$$

Run = 23

$$
12 \\
12 45 \\
12 45 64
$$

Run = 23, 45

$$
12 \\
12 19 \\
12 19 64
$$

Run = 23, 45, 64

$$
12 \\
12 19 \\
12 19 82
$$

Run = 23, 45, 64, 82

$$
12 \\
12 19 \\
12 19 97
$$
Run1 = 23, 45, 64, 82, 97

   12
   12 19
12 19 44

Run2 = 12, 19, 44

#I/OS:

   Read R+S once (seq)
   Write R+S once (seq)
   Read R+S once (random -- can do better if we read long runs instead of
                  1 page at a time, but that requires memory! Might be better to
                  hierarchically merge sequential runs.)

Simple hash:

   i = 0;
   pass size = v (e.g., v = 1) // if P partitions, hash into [1…n], e.g., h(x) = x mod P
   for partition i (on hash values in range range [v*i , v*(i+1)) )
      scan S, hash, if in partition, insert into hash table
      o.w., write back out

      scan R, hash, if in partition, lookup in hash table, output matches
      o.w., write back out

repeat with reduced R and S, in round i+1 ; example:

R = 1, 4, 3, 6, 9, 14, 1, 7, 11
S = 2, 3, 7, 12, 9, 8, 4, 15, 6
h(x) = x mod 3, pass size = 1
Pass 1: h(x) in range [0..1)
R hash table: 3 6 9
remainder: 1 4 14 1 7 11
S probe with : 3 12 9 15 6 --> 3 6 9 join
remainder: 2 7 8 4

Pass 2: h(x) in range [1..2)
R hash table: 1 4 1 7
remainder: 14 11
S probe with : 7 4 --> 7 4 join
remainder: 2 8
Pass 2:  \( h(x) \) in range [1..2)
R hash table: 14 11
S probe with : 2 8 --> no join

Somewhat complex to analyze:

Read R, S (seq)

Amount we write depends on number of passes. In pass 1, we write:

\( ((p-1)/p)R, ((p-1)/p)S \) (seq)

We then read all this data back in (seq), and in pass 2, we write:

\( ((p-2)/p)R, ((p-2)/p)S \) (seq)

And so on...

So for 2 passes, we get:

Read IR+IS, Write (1/2)(IR+IS), Read (1/2)(IR+IS) and are done.
Total IO is 2(IR+IS)

For 3 passes, total IO is 3(IR+IS)
For n passes, total IO is n(IR+IS)

**Grace hash:**

choose P partitions, with one page per partition
hash r into partitions, flushing pages as they fill
hash s into partitions, flushing pages as they fill
for each partition p
    build a hash table T on r tuples in p
    lookup each s in T outputting matches

example:

\[ R = 1, 4, 3, 6, 9, 14, 1, 7, 11 \]
\[ S = 2, 3, 7, 12, 9, 8, 4, 15, 6 \]
\[ h(x) = x \mod 3 \]

\[ R0 = 3 \ 6 \ 9 \]
\[ R1 = 1 \ 4 \ 1 \ 7 \]
\[ R2 = 14 \ 11 \]

\[ S0 = 3 \ 12 \ 9 \ 15 \ 6 \]
S1 = 7 4
S2 = 2 8

Now, join R0 with S0, R1 with S1, R2 with S2

Because we are using the same hash function for R and S we can guarantee that the only tuples that will join with partition Ri are those in Si

How do I pick the partition size?

(Assume uniform distribution of tuples to partitions, make each partition equal to M pages (minus a couple for active pages of S being read.))

\[ \sqrt{|R|} < M \]

partition size = \( M > \sqrt{|R|} \)

#parts \( P = |R|/(M) \leq |R|/\sqrt{|R|} = \sqrt{|R|} \)

\( h(v) \Rightarrow [1,k] \)
each covers \( k/P \) hash values

Need \( \sqrt{|R|} \) pages of memory b/c we need at least one page per partition as we write out (note that simple hash doesn’t have this requirement)

I/O:

read R+S (seq)
write R+S (semi-random)
read R+S (seq)

also \( 3(|R|+|S|) \) I/OS

What's hard about this?

Possible that some partitions will overflow -- e.g., if many duplicate values
What do they say we should do?
(Leave some more slop (assign fewer values to each partition) by assuming that each record takes a few more bytes to store in hash table.)
Split partitions that overflow

When does grace outperform simple?
(When there are many partitions, since we avoid the cost of re-reading tuples from disk in building partitions)

When does simple outperform grace?
(When there are few partitions, since grace re-reads hash tables from disk)

So what does Hybrid do?

\[ M = \sqrt{|R|} + E \]

Make first partition of size \( E \), do it on the fly.
Do remaining partitions as in grace.

Why does grace/hybrid outperform sort-merge?
CPU Costs!
I/O costs are comparable
690 / 1000 seconds in sort merge are due to the costs of sorting
17.4 in the case of CPU for grace/hybrid!

Will this still be true today?
(Yes)